Definitions: Supervaluations

Philosophical Logic 2025/2026

1 Supervaluations

1.1 Models

Definition 1.1 (Model). *A* model *is a pair* $M = \langle V, R \rangle$ *with* $V \neq \emptyset$, $V \subseteq \{0,1\}^P$, *and* $R \subseteq V \times V$. *Thus each element* $v \in V$ *is itself a classical valuation* $v : P \to \{0,1\}$.[1]

1.2 Satisfaction

Let P be a set of propositional variables. For a model $M = \langle V, R \rangle$ and $v \in V$:

$$\begin{array}{lll}
M, v \vDash p & \text{iff} & v(p) = 1 \\
M, v \vDash \neg\phi & \text{iff} & M, v \nvDash \phi \\
M, v \vDash \phi \wedge \psi & \text{iff} & M, v \vDash \phi \text{ and } M, v \vDash \psi \\
M, v \vDash \phi \vee \psi & \text{iff} & M, v \vDash \phi \text{ or } M, v \vDash \psi \\
M, v \vDash \phi \to \psi & \text{iff} & M, v \nvDash \phi \text{ or } M, v \vDash \psi \\
M, v \vDash \Delta\phi & \text{iff} & \forall v' \in V\ (\, vRv' \Rightarrow M, v' \vDash \phi\,).
\end{array}$$

Supertruth

$$M \vDash^1 \phi \quad :\Longleftrightarrow \quad \text{for all } v \in V\ (M, v \vDash \phi).$$

We write $M \vDash^1 \Gamma$ to mean $M \vDash^1 \gamma$ for all $\gamma \in \Gamma$.

1.3 Global Consequence

$$\Gamma \vDash_g \phi \quad \text{iff} \quad \text{for all models } M\ (M \vDash^1 \Gamma \ \Rightarrow\ M \vDash^1 \phi).$$

1.4 Local Consequence

$$\Gamma \vDash_l \phi \quad \text{iff} \quad \text{for all models } M\ \forall v \in V\ (M, v \vDash \gamma \text{ for all } \gamma \in \Gamma \ \Rightarrow\ M, v \vDash \phi).$$

[1] This presentation identifies each "world" with a classical valuation $v : P \to \{0,1\}$. It is more standard to distinguish explicitly between frames and models: a *frame* is $F = \langle U, R \rangle$ (with $U \neq \emptyset$), and a *model* is $M = \langle F, \pi \rangle$ with $\pi : U \times P \to \{0,1\}$. Then the atomic clause is $M, u \vDash p$ iff $\pi(u,p) = 1$, and the determinacy clause is $M, u \vDash \Delta\varphi$ iff $\forall u' \in U\ (uRu' \Rightarrow M, u' \vDash \varphi)$. The current "worlds-as-valuations" setup is recovered by taking $U = V$ and $\pi(u,p) = u(p)$.

1.5 Subtruth and Subfalsity

Definition 1.2 (Subtruth & Subfalsity). *For a model $M = \langle V, R \rangle$ and formula ϕ:*

$$\begin{array}{llll} \text{(Subtruth)} & M \vDash^{\exists 1} \phi & \iff & \exists v \in V : M, v \vDash \phi, \\ \text{(Subfalsity)} & M \vDash^{\exists 0} \phi & \iff & \exists v \in V : M, v \nvDash \phi. \end{array}$$

1.6 Subvaluationist Consequence

Definition 1.3 (Global subvaluationist consequence).

$$\Gamma \vDash^{\exists}_{g} \phi \quad \textit{iff} \quad \textit{for all models } M \ (M \vDash^{\exists 1} \Gamma \ \Rightarrow \ M \vDash^{\exists 1} \phi)$$

where $M \vDash^{\exists 1} \Gamma$ abbreviates $M \vDash^{\exists 1} \gamma$ for all $\gamma \in \Gamma$.